Web Based Interactive Mathematics Modeling System

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Abstract

Cognitive tools are instruments integrated into a learning environment to provide representational support to learners and allow them to focus on the subject matter (van Jooligan, 1999). On the other hand of the scope user modeling started-off as a supporting module to Intelligent Tutoring System allowing them to adapt themselves to student progress. This paper presents a cognitive tool that supports students who learn how to solve mathematical series problems by providing two main features. The interactive feature of the system is able to recalculate the series according to some user specification. On the other hand, the second feature is an open modeling component that allows student to view their models while they are being advise on how to avoid their errors. The combination of the two has resulted in a highly significant increase in student levels from their levels before using the system.

Introduction

"Cognitive Tools" (van Jooligan, 1999) are capable of supporting learners by explicitly representing information. They allow learners to see the structure of the cognitive process by externalizing it and freeing memory for the more important learning task at hand. The simplest form of a tool is a pen and paper, where students can write notes to remind them of the numbers involved when performing addition. Therefore it should not be surprising that computer based educational systems impose themselves at the top of the list of Cognitive Tools. Intelligent Tutoring Systems impose themselves at the top of the list of Cognitive tools because they are capable of surrounding students with an environment that is not rigid in what it offers. One of the potential advantages that it may offer is interactivity. Lawrence, Badre and Stasko (1994) found that students who are exposed to an interactive lab session where they are allowed to create their own graphs and observe how the algorithms works on those graphs did significantly better than those who only attended a classroom lecture. They also found, that those who participated in the active laboratory session did better than those who were passively shown an animation of the algorithm. An interesting point to make here, is that this improvement was not evident in declarative questions, instead it was only clear in procedural questions.

This indicates that student learning of any procedural topic is highly influenced by the presentation style of the teaching material and whether or not it is well suited to the topic under consideration. However, this does not imply that students are not highly individual in nature, and would benefit from a system that reflects their individuality. This belief resulted in the development of user modeling as a field. It aims at presenting students with the right types of material at the right point in time in the right presentation style (Fischer, 2001). This necessitated the existence of a "model" that describes student characteristics at least with respect to a particular task.

"Tutor: What is the integral with respect to x of
$x^4 / (1 + x^2)$
Student: $x + x^3/3 + \tan x$
Tutor (thinks: how did she get that?):…"(Self, 1990)

In order to respond to that question, student modelling systems followed two main approaches. The first, attempted to delve into the cognitive workings of the student's mind and try to best explain how the results could be obtained. Some of those who followed this approach are, Martin & Vahn Lehn (1995), Langley, Wogulis & Ohlsson (1990), Ikeda, Kono and Mizoguchi (1993) amongst others. The second approach assumed that the process that occurs between the "inputs" and "outputs" are in a "black box" scenario. The researchers who adopt this presumption attempts to formulate a mapping between the situation and student response to that situation. Some of those who are following this type of modelling include Webb, Cumming, Richard and Yum (1991) and Webb & Kuzmycz (1996).

Why Participative Learner Modelling?

Those who follow the first approach are in a sense predicting possible causes for student behavior. In order to be able to check the accuracy of the student model in representing the student's cognitive characteristics Vahn Lehn and Niu (2001) conducted a study in sensitivity analysis. They arrived at several interesting conclusions on the factors that influence assessment accuracy. An Intelligent Tutoring System interface for example, seemed to result in a less accurate student model than a Computer Aided Instruction Interface. They also found that the accuracy of the model, strongly depends on what the student is allowed to apply during the course of study because the system can only detect knowledge that is being applied, not knowledge that they may have. This shows a limitation to a student modeler in that it is unable to delve into the student's cognitive structure to obtain any more information than is available through interaction.

"When a learner is engaged in a discussion about the learner model, he is reflecting upon his domain knowledge and experience re-calling and re-considering ideas of which he is aware."(Dimitrova et al, 2000)

The idea from this externalization process is to provide users with the ability to question the assumptions they made about themselves. The existing approaches for involving the learner in the modeling process include open learner models (Paiva and Self, 1995), collaborative student models (Bull et al, 1995) and interactive diagnosis (Dimitrova et al.,2000).

TAGUS is a workbench for dynamic learner modeling (Paiva and Self, 1995), aimed at externalizing and dynamically changing the learner model. The model itself is represented by Prolog clauses and learners can manipulate the model presented by selecting options and typing Prolog clauses in the control panel. In this system, both the educational system and the learner are external agents and interact with the model. Even though the system externalizes the student model, students find a difficulty in understanding and interacting with this representation.

Mr Collins (Bull et al, 1995) is a student model that is open for inspection and negotiation with the student. The system and learner are allowed to have separate and possibly even different perspectives of the students knowledge and argue when they disagree with each other. Here the student model is externalized in tables, which contain domain rules, and the system and learner's measure and assessment confidence about the level of acquisition the learner has for each rule. The communication environment is text based as selections made from menu options. This may prove itself to be confining to a learner's reflection.

STYLE-OLM (Dimitrova et al.,2000) is an interactive diagnosis based modeler in a scientific terminology learning environment. A dialogue game model is suited for maintaining an interactive diagnostics dialogue and diagrammatic communication language provides a graphical externalization of the learner's beliefs. Communication is organized as an exchange of speech acts where dialogue moves are extracted from a framework for analyzing educational dialogues. However, the system itself has not been fully evaluated.

Modeling as a Cognitive Tool

Morales, Pain and Ramscar (1998) distinguish two basic types of representation for the learner's cognitive state. Declarative representation, is a flexible way of storing knowledge that can be accessed for several applications and can be communicated. The second is Procedural representation, which encodes "execution" knowledge that is difficult to describe verbally. The idea is that discussing the model and/or actively participating in altering it, encourages students to verbalize their beliefs about their knowledge. This would turn, the "procedural" knowledge they were following in the task into "declarative" knowledge that is easier to access and use as justification. The authors indicate that the modeling process must be analyzed with respect to viability, efficiency, and usefulness. Yet, it seems that testing for cognitive impact of this type of modeling is not such a straightforward task. The task studied by Morales, Pain and Ramscar (1998) is a cart and pole task, which has clear-cut differences between the procedural and declarative aspects of it. Procedural representation would be of use when a learner is involved in the task itself while declarative when reflecting on whether to go right or left.

Another possible effect of externalizing student models is that of reducing cognitive load. A Cognitive Tool is a tool that supports a student by externalizing enough information to allow the student to concentrate on the more important learning task at hand. Research into this, has shown that Multi-Media Tutoring Systems as an example, offer strong support to student's cognitive load. For example, in the case of Data Structures (Albalooshi & Alkhalifa, 2001) recorded improvements of up to 40% from post classroom levels. This raises an important question; if cognitive load can be supported through multi-media towards learning, then can a dynamic reproduction of their errors help in remediation?

Mirror Modeler

The model was developed using BM's Java Visual Age for Java, which is an integrated visual development environment that facilitates the generation of complex functions. Its main features include the ability to import Graphical User Interfaces (GUIs) and Java Beans that could be constant throughout several applications. The tool generates java applets as in the case of this project or Java Servlets as is required.

The Problem: Mathematical Summation

The Mirror Modeler was set up to teach Mathematical Summation which is a usually challenging topic to students. The summation is usually represented using a Greek symbol, Sigma ($\sum$) and represents the process of adding up the terms in a series. For example, the summation of the series:

$$\sum_{N=1}^{6} \mathbf{N = 1 + 2 + 3 + 4 + 5 + 6}$$

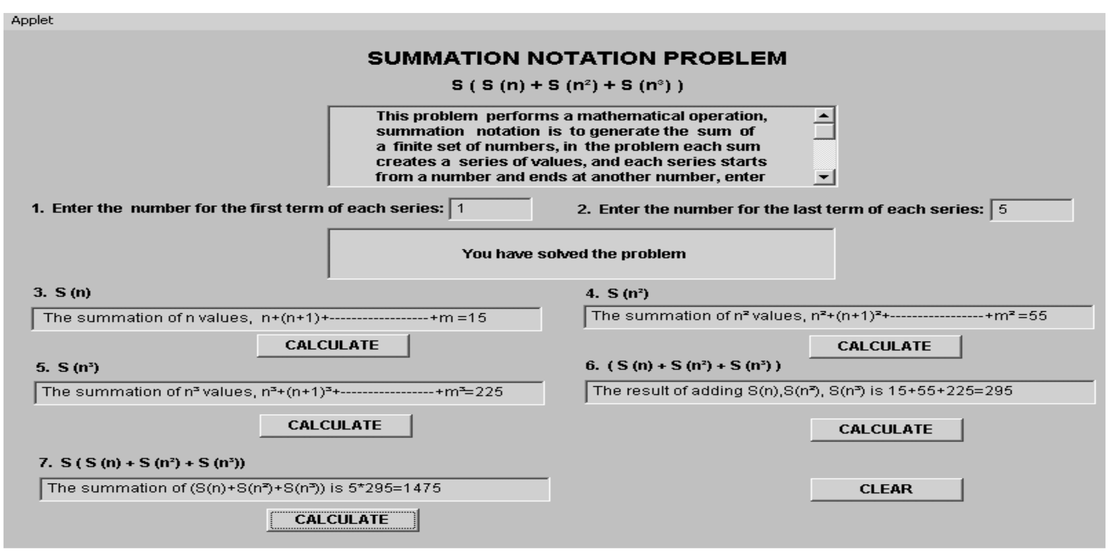
Teaching can be in two directions; either giving students the Summation Notation and asking them to expand it giving the numbers on the right, or giving them the numbers on the right and asking them to return the Summation Notation. The second task is of course, much more challenging than the first.

System Design

The system is composed of a tutorial section, a practice test section, test section, and a model comparison section.

Tutorial Section

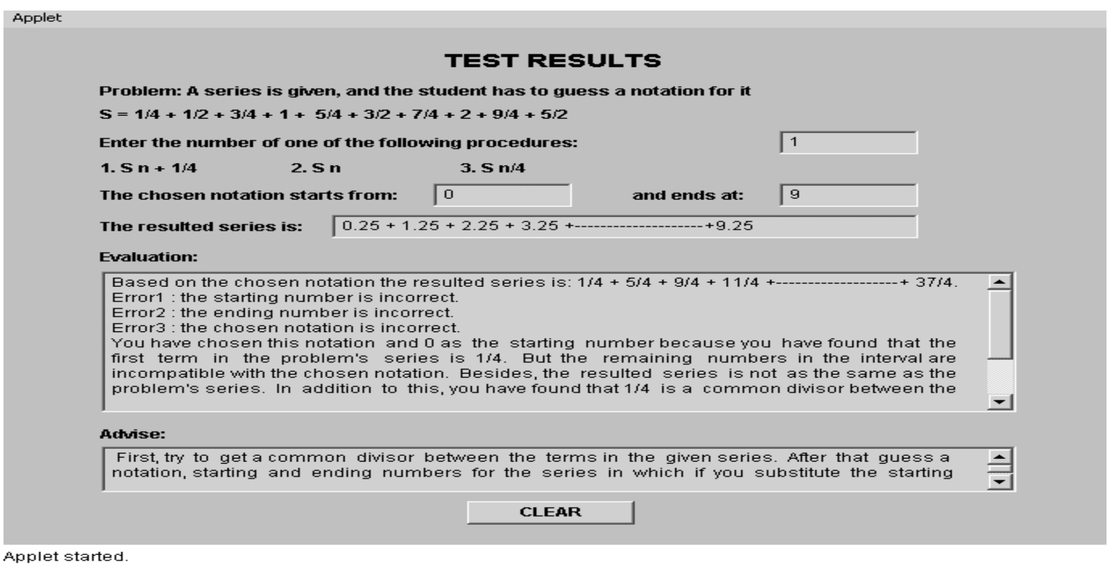
The first section of the system is composed of two main parts that introduce students to the concept of mathematical series by taking them through three examples where they generate the series from the summation notation and seven examples where they are shown how the summation notation is derived from the series. The system is interactive in that it allows students to select some of the variable values and generates the series accordingly whenever possible.
One of the examples where students are shown how to generate the series from the summation notation is shown in figure 1.



Note that the problem is a complex one composed of three terms and is broken up into several parts that are calculated dynamically. Students are allowed to specify the starting and ending terms indicating the length of the resultant series and to be able to recognize how the series can change based on different starting and ending numbers.
The second part of the tutorial, is composed of seven examples to the more difficult task of extracting the notation from the series. The tutorial includes the steps to first select the starting and ending point followed by finding a common divisor and then the generation of the terms of the series to check that it is correct. It is difficult here to allow for student flexibility because the problems given are set problems and its primarily a teaching tool at this stage even though all calculations are still done online.

Practice Test Section

This section is concerned with a more interactive practice session where students write the summation notation they believe to be the answer and are shown the resulting generated series. They can then compare this series to the original and practice any number of times they wish.



At this stage students can select from the different given notations and are allowed to practice and see the result of each selection. They are also given advice of the probable cause of error based on the error made. The number of options varies from one problem to the next to test for student learning and to expose students to more than one possible option.

Test Section

This section is similar to the above in that it has test questions given to students except that here, students are not shown the resulting series so they are not aware of whether or not their answers are correct. Students are showed three problems and they have to fill in several slots with the answers they believe to be true. In a sense, they break up the notation in a starting number, ending number etc to allow the system to dynamically evaluate their responses. Student responses, are then analyzed using production rules that were specifically designed based on a field study of possible student errors in this task.

Model Comparison Section

The student modeling component utilizes simple Bayesian rules to extract the probability of that student makes each type of error and it generates a descriptive verbal model with the results.   The types of errors that are studied are as follows:

Error 1 The arithmetic operation in the chosen notation is incorrect.

Error 2 The integer number in the notation is incorrect.

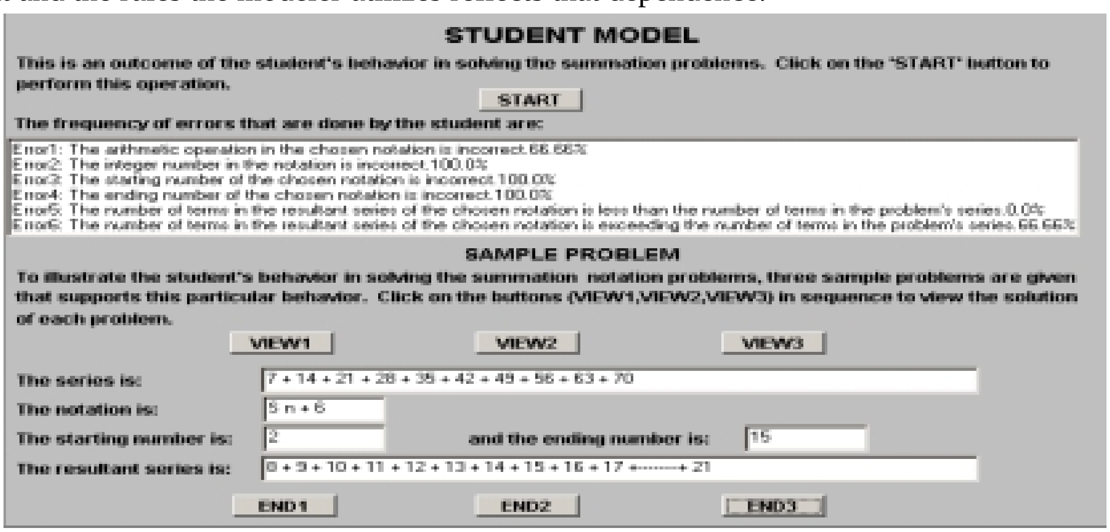
Error 3 The starting number of the chosen notation is incorrect.

Error 4 The ending number of the chosen notation is incorrect.

Error 5 The number of terms in the resultant series of the chosen notation is less than the number of terms in the problem's series.

Error 6 The number of terms in the resultant series of the chosen notation is exceeding the number of terms in the problem's series.

Note that Errors 5 and 6 also depend on Errors 3 and 4, which implies that they are not completely independent and the rules the modeler utilizes reflects that dependence.



The modeler then shows students the idea solution of each of the sample problems while regenerating how they would solve it using their models as a guide. The idea is to compare their behavior to that of the ideal and allow them to reflect on the causes of their errors.

Experiment

In order to understand the effectiveness of having an interactive user interface and an open student model, an experiment was performed to evaluate the amount and areas of student learning that occur in a controlled environment. Therefore, an experiment was performed using a pre and post test that are comparable in questions.

Design

Students were given a paper and pen test that was composed of three questions that tested for three types of series. They solved the questions and were then asked to go through the system's different sections and then asked to solve another three-question paper and pen test that is analogous to the previous three. The aim of this was to find out which questions the system would benefit students in and to compare their error types to the student model produced.

Subjects

12 students from the University of Bahrain participated as volunteers in return for course credit.

Materials

The questions used were specifically selected such that they relate to each other in a way that could be later compared for further analysis. Students would be given the following series of numbers and asked to reproduce the summation Notation that is to the left of each series shown.

Pretest Series

$\sum_{i=1}^{10} i/4$	S=1/4 + 1/2 + 3/4 + 1 + 5/4 + 3/2 + 7/4 + 2 + 9/4 + 5/2
$\sum_{i=1}^{10} 2^i$	S = 2 + 4 + 8 + 16+ 32 + 64 + 128 + 256 + 512 + 1024
$\sum_{i=2}^{11} 3i$	S= 6 + 9 + 12 + 15 + 18 + 21 + 24 + 27 + 30 + 33

Post-test Series

$\sum_{m=3}^{12} 11m$	S=33 + 44 + 55 + 66 + 77 + 88 + 110 + 121 + 132
$\sum_{m=2}^{11} 3^m$	S = 9 + 27 + 81 + 243 + 729 + 2187 + 6561 + 19683 + 59049 + 177147
$\sum_{m=1}^{10} m/7$	S= 1/7 + 2/7 + 3/7 + 4/7 + 5/7 + 6/7 + 1 + 8/7 + 9/7 + 10/7

The mapping between questions types is as follows:

- **Question Type One** is Q1 in the Pretest and Q3 in the Post-test
- **Question Type Two** is Q2 in the Pretest and Question 2 in the Post-test
- **Question Type Three** is Q3 in the Pretest and Question 1 in the Post-test.

Results

Analysis of student responses showed in general that the number of errors made in the Pretest were 37 and the number of errors made in the Post-test were 17 with a probability of **p** <.001 of this happening by chance. Therefore, the system is capable of teaching students how to solve their mathematical series problems.

A more detailed study was made as per question type showing that students did better following using the system for Question Type One and Question Type Three, with **p** < .01 and **p** < .0000 respectively. On the other hand, students did worse in Question Type Two with **p** < .001. This raises a number of questions about the cognitive differences between the operations. Students were much better able to learn how to solve the questions with multiplication and division following the use of the system, while they did much worse in the problems that involved a "power" operation.

General Discussion

The field is open and with a great deal of interesting questions especially with respect to mathematics. Simple operations such as addition, multiplication and power when placed in a series result in a big difference in student performance. Two of these seem to be learnable while the other, not.

The system tested here offers some answers to the questions raised earlier and raises a few new ones. First of all, a complex task as the one presented here, can be taught to students through the mirror modeler to achieve a significant improvement. Although learning is shown through the decrease in the number of errors, the measure used here fails to identify the influencing factor that caused it. Perhaps the units of the system complemented each other or one was redundant. This can only be known through more detailed testing.

The 'power' problem though is an anomaly in that students learn a great deal in all other types but end up significantly worse in this one. It is possible that informing students of their models increases cognitive load to a level that overburdens them when dealing with a challenging problem while helping them when the task requires less of a cognitive load. Much work remains to be done.

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