THE "MEANING" OF STRUCTURE IN DIGITS

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ABSTRACT

A simple dictation task composed of a random order of digits in both the Arabic and English language shows that children learning these digits map the structures of **7** and ٧ making them confuse the directions these digits face. A Kohonen Self Organizing Feature Map was used to represent each digit as a vector and assess their 'closeness' by competition but it failed to explain this behavior. A tensor based Kohonen map was designed to represent digits as matrices and seems to result in a relatively better representation. However, this too fails to explain the behavior, which implies that the behavior can only be explained through the 'meaning' of digit structure. When children learn how to write a digit, the direction it faces seems to be a 'semantic' value associated with the digit and is not described as part of the structure but is "inferred" from it.

1. INTRODUCTION

The current view to the analogical process has recently diverged from the classical view that an inherent distinction between the analogical process and categorization can easily be defined [9]. A direct implication is that a task of English or Arabic digits would not be considered as isolated from each other into two categories. However, although structure does seem to have a strong effect, "semantics" also seems to come up a likely counterpart.

The structural composition of a sentence can affect its meaning even if most of the words are kept constant as in, "Stars shine light in the night" and "Lights shine on the night star". A clear difference in meaning is observed while the words are almost identical with only the structure different. Then it should not at all be surprising to identify that no "structural" representation is void of semantics and no "semantic" representation is void of structure. In the example meaning is implied by the order or direction.

However, to date, a great deal of emphasis has been placed on the importance of structural similarity alone and its role in the analogical process [7], [3]. Perhaps within the same "structural" framework some semantics may be uncovered. The secret of the interplay between the two, cannot be found in either "structure" alone or "semantics" alone, so it may very well be a trait of the cognitive mechanism used for analogies.

> ***"We talk of process and states, and leave their nature undecided. Sometime perhaps we will know more about them – we think. But that is just what commits us to a particular way of looking into the matter." [12]***

Alas, we find the common thread. It is our commitment to "a particular way of looking into the matter". So the only way we could deal with such a highly diverse group of exemplars is to look at matters with a particular "directionality of thought". Freyd [4] through her theory of Dynamic Momentum, showed through a large medly of dispersed experiments, that there does seem to be not only a "directionality of thought" but that it could be influenced through even static stimuli. In one of the experiments [2], she showed subjects a series of static images representing a rectangle that is rotating. She then asked them to choose one of several positions to select the last position they saw. She found a significant bias towards selecting the position that followed from the last one they saw in sequence. This series of images may be regarded as implying motion as with children's cartoons. Therefore, a safe conclusion to make here is that mental representational is somehow 'directed' towards the next step, so it finds it easier to accept then the actual last position shown.

2. THE MAPPING OF "DIRECTION"

In the context of the Wason Selection Task [11] a test of direction was designed showing subjects four diagrams of conveyor belts that carry either a striped or a grey object [1]. Students were given a rule to test, <u>If the striped cube is in the conveyor belt system, then the grey cylinder must be the grey object directly following it as the conveyor belt moves.</u> They have to select the least number of cases in which they would open one of the boxes to see the other object. This question was placed either before or after the classical Wason Card task [11] where subjects are shown four cards; e.g. with A B 4 and 7 and asked which cards they would turn to check this rule. If A is on one side of the card, then 4 must be on the other side of the card. The correct answers in the card task are A and 7. When students are given clear wording that the conveyor was moving, their selections seemed to be more directed towards not selecting P if the rule is IF P THEN Q. If they not given any clear implication that the conveyor is not moving there selections of NotQ seems to be strongly affected. In both cases, there seems to be a strong effect of directional alignment of the semantics of the rule, to the rule itself. Additionally, in spite of the limited nature of structural alignment of the two tasks a carry-over effect from one to the other did exist. In order to study this effect a much simpler task has to be considered.

> ***"Their parable is like the parable of one who kindled a fire but when it had illumined all around him, Allah took away their light, and left them in utter darkness-- they do not see." (Koran,2:17)***

Even in religious texts, we find a strong use of analogies in a diverse setting of topics, which if anything should indicate that this "mode of thinking" does seem to be well founded. With children though, there seems to be a "special case" where Goswami [5] shows evidence that analogical reasoning abilities appear early in a child's age and plays a crucial role in learning. For example, five and six year old children taught to read a word like ***beak*** can use the relation between the spelling and sound as a basis for analogies about the spelling and pronunciations of new words like ***peak***. Ratterman and Gentner [10] provide more evidence for a relational shift in the development of analogy in children. They claim that children interpret analogies and metaphors first in terms of object similarity and only later in terms of relations to each other. It seems that the question of whether or not children are able to utilize analogies as a way of predicting ambiguous issues is not an issue as clear evidence exists that they use analogies in learning. The problem, however, seems to be how can the features of a shape be delineated in a study to review their influence and what is the "meaning" behind this shape if no clear-cut relations are implied or assumed?

3. EXPERMENT ONE

The aim of this experiment is to detect any existing anomalies in the way children write digits in both the Arabic and English language. The task is a dictation of a random order of digits either in both language or in one.

3.1 Subjects

29 children with the average age of 6 years and taught only in Arabic did the Arabic only version of the test. 27 children with the average age of 9 who had been introduced to the English language one year prior did the mixed version of the test.

3.2 Materials

A random number generator was used to randomly order the digits so that all children wrote all digits either 10 digits in the Arabic language scenario or 20 digits in the mixed language scenario.

3.3 Results

6 first graders flipped at least one digit horizontally while 23 wrote their numbers correctly. On the other hand, 10 fourth graders flipped at least one digit horizontally while 17 wrote their digits correctly. The difference caused by the introduction of the English language in grade 3 was significant with $\underline{\mathbf{p}} < .0297$.

It seems that the Arabic language alone caused a strong effect and the introduction of the English language, simply added to the frequency of the error as is expected by introducing a new set of digits. Therefore, we can safely assume that the effect is language independent. This result offers strong support to that associating numbers is done through analogical mappings rather than their classification into two distinct languages.

4. A KOHONEN SELF ORGANIZING FEATURE MAP

This is a neural network algorithm designed by Tuevo Kohonen [8] that allows the inputs to organize themselves in a high dimensional space as per a competition.

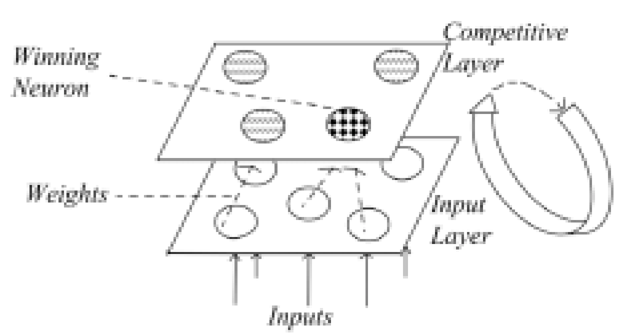


Figure 1: A Kohonen Network Architecture

Input patterns are presented as vectors through a training phase and each is classified by the units it activates based on its similarities to other members of that group. These similarities are mapped into 'closeness' measure that is dynamically estimated by the competitive layer. The networks used here are unsupervised in the sense that they are only given the inputs and allowed to search for 'closeness' measures between the different inputs. The decision of how close any input is, must be assessed through a special function that is different according to the net representation.

5. VECTOR BASED MODEL

A Kohonen map was designed specifically for this purpose.

5.1 Inputs

Its inputs were composed of 20 digits, ten Arabic digits from 0 to 9 and ten English digits from 0 to 9. These digits were represented on a 5x5 grid and treated as a binary input of either value 0 or 1 per grid cell. Since the test here is a comparison between digit structures, a basic form was used and not a handwritten representation. Additionally, weight vectors are five in number so that the input digits could be grouped together according to similarity. The weight vector is initialized with a random number generating function.

5.2 Net Design

The net itself, accepts these inputs one at a time, as a vector representation in 25 dimensions. It searches for the closest weight vector to the input. The function used is as follows:

Subtraction

$$\underline{v} - \underline{w} = \sum_{i=1}^{25} v_i - w_i$$

Magnitude

$$|\underline{v}| = \sqrt{\sum_{i=1}^{25} v_i^2}$$

The 'closest' weight vector is determined by a measure of subtraction between the weight vectors and the input vector. This is then sent through a competition function that ascertains which is the lowest value in the group. If two values or more are equal as the least, a random function is used to generate a seed and determine the winner.

Once the net locates the 'closest' weight vector, it adjusts that weight vector according to a predetermined learning rate. The learning rate ensures that the difference imposed on the weight vector is a merely a small step towards getting it closer to the input vector. This is necessary so that the weight vector is adjusted so as to generalize for several digits that share 'shape similarity'.

Each time the full set of inputs is given to the net, the weights are adjusted only according to how close they are to these inputs. Therefore, there is never any guarantee or way of prior bias of which digits end up to seem closer to each other.

5.3 Results

A summary of the correlation values for the structure of English digits from 1 to 0 is showed in the following table with an average of 0.4783.

	1E	*2E*	*3E*	*4E*	*5E*	*6E*	*7E*	*8E*	*9E*	*0E*
1E	0									
2E	1.3	0								
3E	.98	.32	0							
4E	1	.43	.33	0						
5E	1.5	.24	.49	.63	0					
6E	1.3	.24	.40	.51	.22	0				
7E	.88	.46	.21	.39	.65	.58	0			
8E	1.2	.22	.30	.44	.26	.11	.49	0		
9E	1.1	.29	.22	.26	.43	.33	.39	.24	0	
0E	1	.47	.33	.49	.56	.38	.48	.34	.41	0

Table 1: Correlation values of English Digits

A summary of the correlation values for the structures of Arabic digits from 1 to 0 is shown in the following table with an average of 0.7138.

	1A	*2A*	*3A*	*4A*	*5A*	*6A*	*7A*	*8A*	*9A*	*0A*
1A	0									
2A	.67	0								
3A	.99	.64	0							
4A	1	.41	.52	0						
5A	1	.45	.61	.16	0					
6A	.7	.38	.62	.41	.35	0				
7A	.83	.47	.53	.49	.58	.49	0			
8A	.75	.5	.56	.47	.40	.19	.58	0		
9A	1.1	.56	.67	.22	.17	.45	.60	.51	0	
0A	1.2	1.7	1.7	2	2	1.7	1.6	1.7	2.1	0

Table 2: Correlation values of Arabic Digits

The following table shows the correlation between the structures of Arabic versus English digits with an average of 0.7372.

	1E	*2E*	*3E*	*4E*	*5E*	*6E*	*7E*	*8E*	*9E*	*0E*
1A	0	1.3	.99	1.0	1.5	1.3	.88	1.2	1.1	1
2A	.67	.65	.38	.51	.82	.68	.32	.61	.56	.39
3A	.99	.70	.54	.66	.87	.88	.39	.80	.67	.83
4A	1	.31	.05	.33	.43	.41	.22	.31	.22	.36
5A	1	.40	.16	.33	.52	.42	.33	.31	.17	.37
6A	.71	.69	.40	.39	.86	.72	.40	63	.45	.53
7A	.83	.64	.50	.42	.88	.79	.39	.73	.60	.66
8A	.75	.75	.46	.49	.91	.80	.43	.70	.51	.66
9A	1.1	.29	.22	.26	.43	.33	.40	.24	0	.41
0A	1.2	2.2	2.0	1.9	2.4	2.4	1.8	2.3	2.1	2.1

Table 3: Correlation values of Arabic vs. English Digits

5.4 Discussion

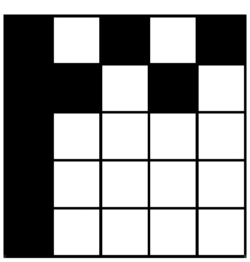
These results indicate that Arabic digits are more dispersed in the vector space than English digits with a variance of 0.3097 while the English digits have a variance of 0.1298. This indicates that the differences in shapes amongst Arabic digits are more than those among English digits. Another amazing result is the 0.05 similarity ratio found between the English number 3 and the Arabic number ٤ which is surprising because they are facing different directions. Although this is done for these two digits, it does not repeat the same behavior for the numbers ٢ and ٦ which should be considered closer than the two above yet they have a similarity ratio of 0.0.3843. To further understand this data, another model was constructed. This time each number represented on a 5x5 grid was used as training input to the Neural Net as a tensor, or as a matrix. This implied that the net design be modified accordingly resulting in a new Tensor based Kohonen architecture.

6. TENSOR BASED MODEL

A Kohonen architecture used above was modified to deal with tensor inputs. In the previous model, the representation was composed of a group of vectors represented in a multi-dimensional space with 25 dimensions. In this model, the representation is of a group of tensors, each of which is represented through a matrix and each is projected into a space and evaluated with respect to their similarity distances, etc.

While a vector associates a group of values with a point, a tensor [6] associates a matrix with a point. By analogy a vector representation, associates a number of values with a digit, and a tensor representation associates a matrix or an ordered set of values, with a digit. This matrix, can emphasize both vertical as well as horizontal information.

The number three for example in Arabic will be a vector formed of the values [1 0 1 0 1 1 1 0 1 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0] that are then normalized to be used as input or as a matrix represented as:



[1 0 1 0 1
1 1 0 1 0
1 0 0 0 0
1 0 0 0 0
1 0 0 0 0]

Figure 2: Mapping from Digit to Representation

Clearly the latter representation preserves the 'closeness' of the cells that are vertically related and this difference in representation is investigated here. The reason this representation was used to get a closer representation to the one that children are required to remember. There is a hope of course to identify the causes of the errors in direction and if these causes lie in the structure of the number itself as is presented or the 'implications' or 'meaning' understood when this structure is presented to children.

6.1 Inputs

This net's inputs were composed of 20 digits, ten Arabic digits from 0 to 9 and ten English digits from 0 to 9 same as the one above with the same coding measures. Here they were also represented on a grid, and coded as a binary input and then normalized.

Additionally, tensor weight vectors are five in number so that the input digits could be grouped together according to similarity. The weight tensors are 5x5 matrices that are initialized with a random number generating function.

6.2 Net Design

Since this neural net architecture deals with tensors, several basic tensor operations were necessary. Each digit was first represented as a matrix rather than a vector and presented to the system. This representation is then compared to each of the weight tensors with respect to distance. The tensor formula used for this estimation is as follows:

$$\underline{\underline{T}} - \underline{\underline{U}} = \sum_{i=1}^{5}\sum_{j=1}^{5} T_{ij} - U_{ji}$$

The result here was the distance between the two and it's a third tensor. The magnitude of this tensor was then estimated using this formula:

$$\left|\underline{\underline{T}}\right| = \sqrt{\frac{1}{2}\sum_{i=1}^{5}\sum_{j=1}^{5} T_{ij}^{2}}$$

This resulted in a scalar value relating each of the weight tensors to the input tensors. The lowest of these was used to select the 'winning' weight tensor. This tensor was then adjusted according to the learning rate. If the competition resulted in two equidistant tensors then the choice would be made through a random function.

Once the adjustment is made the program continues through the loop to complete the full presentation of the whole data set according to the number of epochs.

A special point worth noting here, is that the testing phase needs some programming as well because the tensor dot product is different from matrix multiplication so although MATLAB[1] offers many preset functions, they are not useful in this case. The formula used for the dot product is as follows:

$$\underline{\underline{T}} : \underline{\underline{U}} = \sum_{i=1}^{5} \sum_{j=1}^{5} T_{ij} U_{ji}$$

6.3 Results

A summary of the correlation values for the structure of English digits from 1 to 0 is showed in the following table with an average of 0.6822.

	1E	2E	3E	4E	5E	6E	7E	8E	9E	0E
1E	0									
2E	1.6	0								
3E	1.5	.30	0							
4E	1.5	.97	.98	0						
5E	1.8	.46	.61	.68	0					
6E	1.5	.41	.47	.67	.28	0				
7E	.97	.69	.54	.87	.86	.63	0			
8E	1.3	.58	.31	.93	.78	.61	.43	0		
9E	1.3	.83	.78	.30	.68	.61	.64	.67	0	
0E	1.5	.29	.30	.76	.42	.30	.56	.43	.59	0

Table 4: Results of Tensor Model – English Digits

A summary of the correlation values for the structures of Arabic digits from 1 to 0 is shown in the following table with an average of 1.1084.

	1A	2A	3A	4A	5A	6A	7A	8A	9A	0A
1A	0									
2A	.80	0								
3A	1.6	1.1	0							
4A	1.1	0.4	1.3	0						
5A	1.8	1	.82	1	0					
6A	1.3	.54	.97	.46	.74	0				
7A	1.4	.94	.40	1	.82	.69	0			
8A	1.4	.83	.37	.99	.58	.66	.34	0		
9A	1.3	.61	1	.59	.87	.22	.69	.71	0	
0A	1.1	1.8	2.6	1.9	2.8	2.2	2.4	2.4	2.2	0

Table 5: Results of Tensor Model – Arabic Digits

The following table shows the correlation between the structures of Arabic versus English digits with an average of 1.0107.

	1E	2E	3E	4E	5E	6E	7E	8E	9E	0E
1A	0	1.6	1.5	1.5	1.8	1.5	.97	1.3	1.3	1.5
2A	.80	.83	.72	.85	.98	.77	.24	.59	.61	.68
3A	1.6	1.2	1.4	.86	1	1	1.2	1.4	1	1.2
4A	1.1	.72	.62	.88	.91	.79	.47	.49	.59	.56
5A	1.8	.54	.77	.84	.37	.43	.94	.98	.87	.58
6A	1.3	.65	.65	.44	.58	.52	.56	.59	.22	.44
7A	1.4	1.1	1.2	.54	.90	.88	1	1.2	.69	.98
8A	1.4	.91	1	.62	.73	.66	.86	1.1	.71	.81
9A	1.3	.83	.78	.30	.68	.61	.64	.67	0	.59
0A	1.1	2.6	2.5	2.4	2.8	2.6	2	2.2	2.2	2.5

Table 6: Results of Tensor Model – English vs. Arabic

[1] Copyright, Mathworks Inc.

7. DISCUSSION

The variance differences here can also be seen clearly between the digits used in both languages. The English digits have a variance of 0.1936 and the Arabic digits have a variance of 0.4945. However, the interesting results emerge when we measure the change in relative distances between the digits when apply this modelling technique rather than the first. To be able to compare the two results the averages equated to get a common basis for comparison and the first set of results was adjusted accordingly. The clearest differences appeared in the following sets of differences:

٦ was brought farther from ٢ in the tensor model by 0.64
٥ was brought farther from ٩ in the tensor model by 0.63
3 was brought farther fron ٤ in the tensor model by 0.56
3 was brought farther from 9 in the tensor model 0.47
2 was brought closer to ٦ in the tensor model by 0.31
5 was brought closer to ٦ in the tensor model by 0.61
٢ was kept as far as it was from ٦ in both models.

Notice that the vector model made the same 'mistakes' children make while the tensor model was more faithful to the digit structure. The differences in shape between 3 and are clear especially when it comes to the direction that the letter faces, the similarity is vertical not horizontal. The vector model is based on having horizontal line ٤_scriptions rather than the two combined, therefore it seems to be a little blind to 'direction'.

8. The "Meaning" in Structure

The tensor model compares 3D representations of the digits mapped to a point rather than comparing 2D representations. This is like mapping mountains to each other rather than mapping arches. The extra power was better able to capture the differences because it highlighted differences that were invisible to the vector model and at the same time noticed similarities.

However, the children tested in the experiment described above did make this error and the errors they made increased when the second language was introduced. One possible explanation for it could be that they are indifferent to the direction the digit faces and their memory is unable to retain that information. But how could this be the case when both the vector and tensor models managed to clearly differentiate between ٢ and ٦ while these are the two letters children get mixed up in them most. Additionally, in no language does the digit 9 look like this . Yet so many children did this exact mistake.

This is all good and well, but it fails to explain what we have here. The direction of writing is horizontally flipped and all internal rules between the lines are kept the same so the cited experiment offers no explanation as to why children flip the letters horizontally. Evidently both ways of looking at the letters will allow the any distortions to be consistent with the way of writing.

The tensor model is as detailed a model analysis that any can be developed to study the shape of these two dimensional digits, and this model too does not offer any explanation. This leaves only two choices, either the children's behavior is incomprehensible, or it is caused by an 'implication' that they sense when confronted with these structures. By definition, this implication is 'meaning' and in this case this 'meaning' is inherent in the structure of the digits. The reason the word meaning is in quotes is that this seems to be a special type of meaning that simply describes one aspect of the structure, the direction it faces. In a sense, these results imply that children, living in a three dimensional world seem to be aware that objects in life are the same whether they face the right side or the left. By virtue of comparison, if this is applied to digits, they would assume that any structure that faces a direction, is the same whether it faces right or left. This added piece of information is not inherently represented as part of the structure but instead is assumed through analogy. This analogical process is further applied when digits facing different directions follow each other because the chances of a child writing a digit facing left is higher if the digit preceding it faces left as well.

A good point to ponder at the end of this discussion is that the inventors of the English digits were Arabs as they are also called the Arabic numerals, yet they left them to use the currently called Indian numerals which are the Arabic digits. In a quick study of the direction of digits the following can be clearly seen:

0	1	2	3	4	5	6	7	8	9
		◄	◄	◄	◄	►	◄		◄
٠	١	٢	٣	٤	٥	٦	٧	٨	٩
		►	►	►		◄			◄

Table 7: Directions Numbers face in both languages

Although the English digits were developed based upon the number of angles, which is a sort of mathematical basis, they are mixed up in directions and have a bias towards one. On other hand, the later selected Arabic digits have a more uniform 3:2 ratio, which is more 'balanced' with respect to the direction the letters face. If the meaning in structure is 'direction', why did Arabs abandon the English digits to the more direction-balanced currently used digits? Note that the 7 and 8 in Arabic are vertically flipped yet children rarely get them wrong.

9. Acknowledgement

I would like to thank god for giving me enough faith in myself to see these models through to completion and the stars of the sky for illuminating the dark nights of wonder and confusion.

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